MODULE-2

A.C. circuits

The path for the flow of alternating current is called on a.c. circuit.

In a d.c. circuit, the current/flowing through the circuit is given by the simple relation $I = \frac{V}{R}$. However, in an a.c. circuit, voltage and current change from instant to instant and so give rise to magnetic (inductive) and electrostatic (capacitive) effects. So, in an a.c. circuit, inductance and capacitance must be considered in addition to resistance.

We shall now deal with the following a.c. circuits:

i) AC circuit containing pure ohmic resistance only.
ii) AC circuit containing pure inductance only.
iii) AC circuit containing pure capacitance only.

AC circuit containing pure ohmic Resistance

When an alternating voltage is applied across a pure ohmic resistance, electrons (current) flow in one direction during the first half-cycle and in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

be given by the equation

$\nu = V_m \sin \theta = V_m \sin \omega t$ --- (i)

As a result of this alternating voltage, alternating current „i‟ will flow through the circuit.

The applied voltage has to supply the drop in the resistance, i.e.,

$\nu = iR$

Substituting the value of „ν‟ from eqn. (i), we get

$V_m \sin \omega t = iR$ or $i = \frac{V_m}{R} \sin \omega t$ --- (ii)

The value of the alternating current „i‟ is maximum when sin $\omega t = 1$, i.e., $I_m = \frac{V_m}{R}$

$\therefore$ Eqn.(ii) becomes,

$i = I_m \sin \omega t$ --- (iii)

From eqns.(i) and (ii), it is apparent that voltage and current are in phase with each other. This is also indicated by the wave and vector diagram shown in Fig. 3.32.

Power: The voltage and current are changing at every instant.

$\therefore$ Instantaneous power, $P = V_m \sin \omega t. = I_m \sin \omega t$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{(1-\cos 2\omega t)}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}.\cos 2\omega t$$

Thus instantaneous power consists of a constant part $\frac{V_m I_m}{2}$ and a

Fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of voltage and current waves.

The average value of $\frac{V_m I_m}{2} \cos 2\omega t$ over a complete cycle is zero.

So, power for the complete cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or P = V1 watts

Where V = r.m.s. value of applied voltage

I = r.m.s. value of the current

Power curve

The power curve for a purely resistive circuit is shown in Fig. 3.33. It is apparent that power in such a circuit is zero only at the instants a,b and c, when both voltage and current are zero, but is positive at all other instants. in other words, power is never negative, so that power is always lost in a resistive a.c. circuit. This power is dissipated as heat.

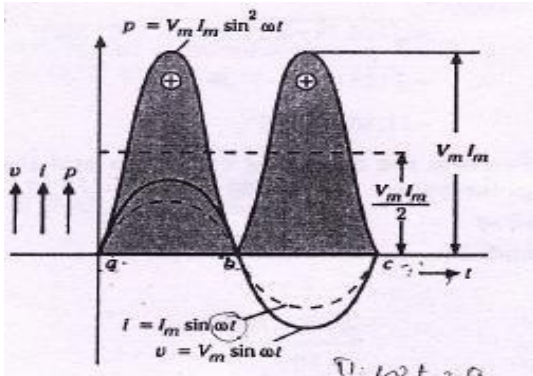


Fig. 3.33

A.C. circuit containing pure Inductance

An inductive coil is a coil with or without an iron core and has negligible resistance. In practice, pure inductance can never be had as the inductive coil has always a small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance, so, for the purpose of our study, we will consider a purely inductive coil.

On the application of an alternating voltage (Fig.3.34) to a circuit containing a pure inductance, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall o f current, at every stage. Because of the absence of voltage drop, the applied voltage has to overcome this self-induced e.m.f. only.

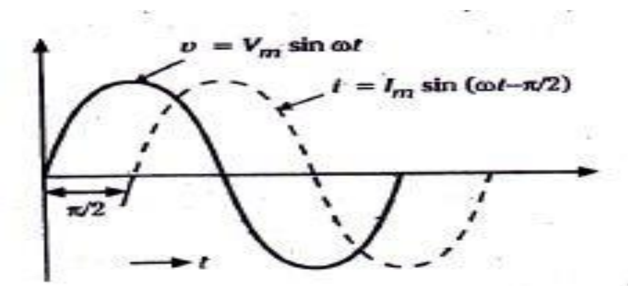


Fig. 3.35

Inductive Reactance: ωL in the expression $I_m = \frac{V_m}{\omega L}$ is known as inductive reactance and is denoted by X_L, i.e., $X_L = \omega L$. If „L" is in henry and „ω is in radians per second, then X_L will be in ohms. So, inductive reactance plays the part the part of resistance.

Power: Instantaneous Power,

$$P = vi = V_m \sin \omega t \,.\, I_m \sin \omega t$$

$$= - V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{-V_m I_m}{2} \sin 2\omega t$$

The power measured by a wattmeter is the average value of „p", which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Put in mathematical terms,

Power for the whole cycle, $P = - \frac{-V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \; dt = 0$

Hence, power absorbed in a pure inductive circuit is zero.

Power curve

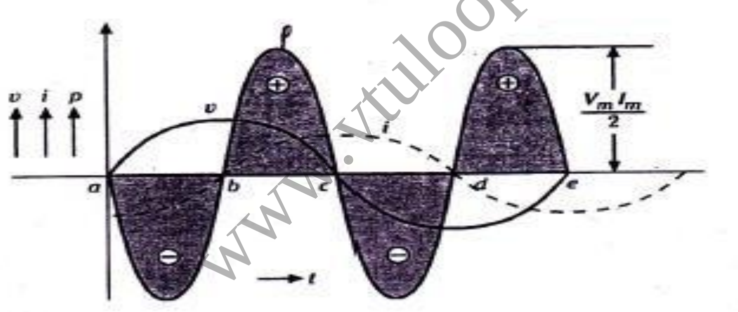


Fig. 3.36

The power curve for a pure inductive circuit is shown in Fig. 3.36. This indicates that power absorbed in the circuit is zero. At the instants a,c and e, voltage is zero, so that power is zero: it is also zero at points b and d when the current is zero. Between a and b voltage and current are in opposite directions, so that power is negative and energy is taken from the circuit. Between b and c voltage and current are in the same direction, so that power is positive and is put back into the circuit. Similarly, between c and d, power is taken from the circuit and between d and e it is put into the circuit. Hence, net power is zero.

AC circuit containing pure capacitance

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the opposite direction as the voltage reverses. With reference to Fig. 3.38,

Let alternating voltage represented by $v = V_m \sin \omega t$ be applied across a capacitor of capacitance C Farads.

Instantaneous charge, $q = cv = CV_m \sin \omega t$

Capacitor current is equal to the rate of change of charge, or

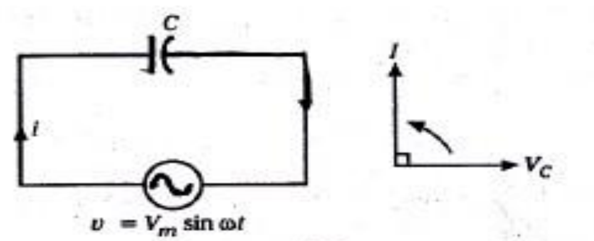


Fig. 3.38

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t)$$

$$= \omega CV_m \cos \omega t$$

or $$i = \frac{V_m}{\frac{1}{\omega c}} \sin \omega t$$

The current is maximum when t = 0

$$\therefore I_m = \frac{V_m}{\frac{1}{\omega c}}$$

Substituting $\frac{V_m}{\frac{1}{\omega c}} = I_m$ in the above expression for instantaneous current, we get

$$i = I_m \sin \omega t$$

Capacitive Reactance: $\frac{1}{\omega c}$ in the expression $I_m = \frac{V_m}{\frac{1}{\omega c}}$ is known as capacitive reactance and is denoted by X_c.

i.e., $X_c = \frac{1}{\omega c}$

If C is farads and „ω" is in radians, then X_c will be in ohms.

It is seen that if the applied voltage is given by $v = V_m \sin \omega t$, then the current is given by $i = I_m \sin \omega t$ this shows that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 3.39, or phase difference between its voltage and current is $\frac{\pi}{2}$ with the current leading.

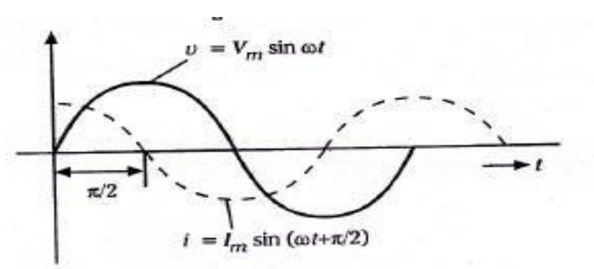


Fig. 3.39

Power: Instantaneous Power,

$$P = vi$$

$$= V_m \sin \omega t . I_m \sin \omega t$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t$$

Power for the complete cycle

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

Hence power absorbed in a capacitive circuit is zero.

Power curves (Fig. 3.40)

At the instants b,d, the current is zero, so that power is zero; it is also zero at the instants a,c and e, when the voltage is zero. Between a and b, voltage and current are in the same direction, so that power is positive and is being put back in the circuit. Between b and c, voltage and current are in the opposite directions, so that power is negative and energy is taken from the circuit. Similarly, between c and d, power is put back into the circuit, and between d and e it is taken from the circuit.

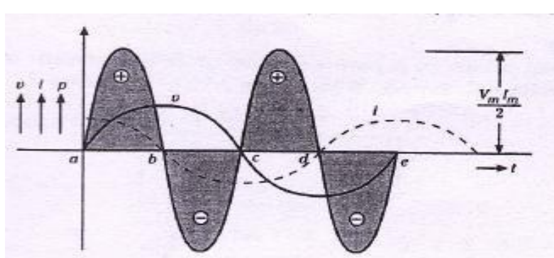


Fig. 3.40

Therefore, power absorbed in a pure capacitive circuit is zero.

Series R-L circuit

Let us consider an a.c. circuit containing a pure resistance R ohms and a pure inductance of L henrys, as shown in Fig. 3.43.

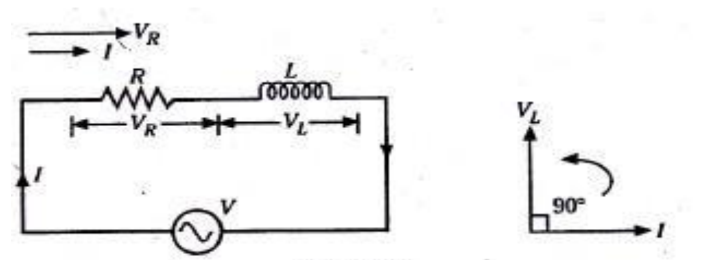


Fig. 3.43

Let V = r.m.s. value of the applied voltage

I = r.m.s. value of the current

Voltage drop across R, $V_R = IR$ (in phase with I)

Voltage drop across L, $V_L = IX_L$ (leading I by 90^0)

The voltage drops across these two circuit components are shown in Fig. 3.44, where vector OA indicates V_R and AB indicates V_L. The applied voltage V is the vector sum of the two, i.e., OB.

$$\therefore \quad V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

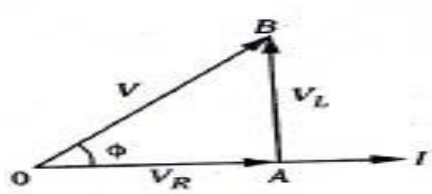


Fig. 3.44

The term $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called the impedance (Z) of the circuit. It is measured in ohms.

$$\therefore \quad I = \frac{V}{Z}$$

Referring to the impedance triangle ABC, (Fig. 3.45)

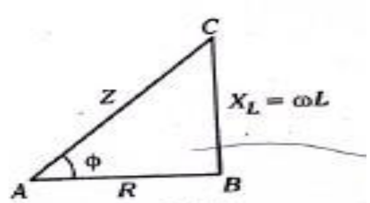


Fig. 3.45

$$Z^2 = R^2 + X_L^2$$

or (impedance)^2 + (reactance)^2

Referring back to Fig. 3.44, we observe that the applied voltage V leads the current I by an angle ϕ.

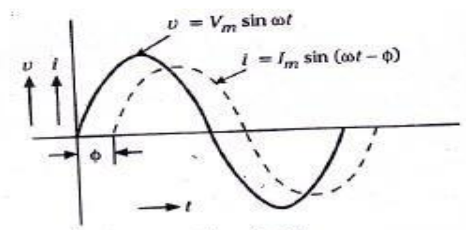


Fig. 3.46

$$\tan\phi = \frac{V_L}{V_R} = \frac{I.X_L}{I.R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

$$\therefore \qquad \phi = \tan^{-1}\frac{X_L}{R}$$

The same feature is shown by means of waveforms (Fig. 3.46). We observe that circuit current lags behind applied voltage by an angle ϕ.

So, if applied voltage is expressed as $v = V_m \sin \omega t$, the current is given by $i = I_m(\sin \omega t - \phi)$

Where $I_m = \frac{V_m}{Z}$.

Definition of Real power, Reactive Power, Apparent power and power Factor

Let a series R-L circuit draw a current I (r.m.s. value) when an alternating voltage of r.m.s. value V is applied to it. Suppose the current lags behind the applied voltage by an angle ϕ as shown in Fig. 3.47.

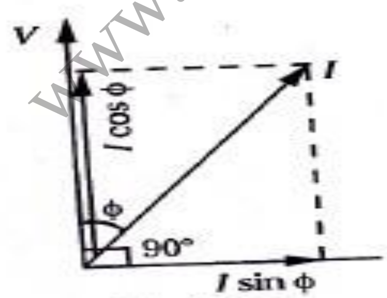


Fig. 3.47

Power Factor and its signifies

Power Factor may be defined as the cosine of the angle of lead or lag. In Fig. 3.47, the angle of lag is shown.

Thus power Factor = cos ϕ.

In addition to having a numerical value, the power factor of a circuit carries a notation that signifies the nature of the circuit, i.e., whether the equivalent circuit is resistive, inductive or capacitive. Thus, the p.f. might be expressed as 0.8 lagging. The lagging and leading refers to the phase of the current vector with respect to the voltage vector. Thus, a lagging power factor means that the current lags the voltage and the circuit is inductive in nature. However, in the case of leading power factor, the current leads the voltage and the circuit is capacitive.

Apparent Power: The product of r.m.s. values of current and voltage, VI, is called the apparent power and is measured in volt-amperes (VA) or in kilo-volt amperes (KVA).

Real Power: The real power in an a.c. circuit is obtained by multiplying the apparent power by the factor and is expressed in watts or killo-watts (kW).

Real power (W) = volt-amperes (VA) × power factor cos ϕ

or **Watts = VA cos ϕ**

Here, it should be noted that power consumed is due to ohmic resistance only as a pure inductance does not consume any power.

Thus, $P = V I \cos\phi$

$\cos\phi = \frac{R}{Z}$ (refer to the impedance triangle of Fig. 3.45)

$\therefore\ P = V I \times \left[\frac{R}{Z}\right]$

$= \left[\frac{V}{Z}\right] \times IR = I^2R$

or $\mathbf{P = I^2R}$ **watts**

Reactive Power: It is the power developed in the inductive reactance of the circuit. The quantity VI sin ϕ is called the reactive power; it is measured in reactive volt-amperes or vars (VAr).

The power consumed can be represented by means of waveform in Fig. 3.48.

We will now calculate power in terms of instantaneous values.

Instantaneous power, $P = vi = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$

$= V_m I_m \sin \omega t \sin(\omega t - \phi)$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$

This power consists of two parts:

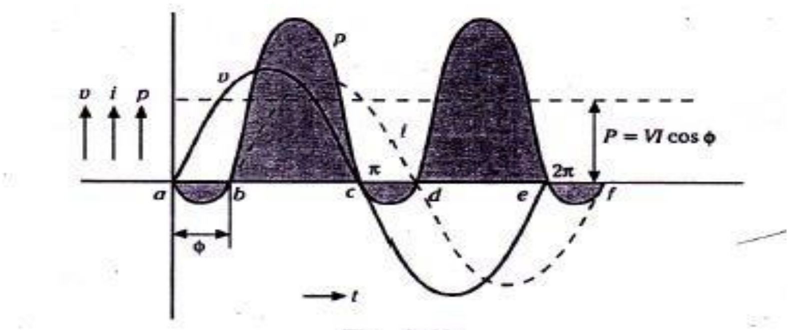


Fig. 3.48

i) Constant part $\frac{1}{2}V_m I_m \cos\phi$ which contributes to real power.

ii) Sinusoidally varying part $\frac{1}{2}V_m I_m \cos(2\omega t - \phi)$, whose frequency is twice that of the voltage and the current, and whose average value over a complete cycle is zero (so it does not contribute to any power).

So, average power consumed, $P = \frac{1}{2}V_m I_m \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\mathbf{= V\,I\cos\phi}$$

Where V and I are r.m.s. values

Power curves:

The power curve for R-L series circuit is shown in Fig. 3.48. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power over the cycle is positive.

During the time interval a to b, applied voltage and current are in opposite directions, so that power is negative. Under such conditions, the inductance L returns power to the circuit. During the period b to c, the applied voltage and current are in the same direction so that power is positive , and therefore, power is put into the circuit. In a similar way, during the period c to d, inductance L returns power to the circuit while between d and e, power is put into the circuit. The power absorbed by resistance R is converted into heat and not returned.

Series R – C circuit

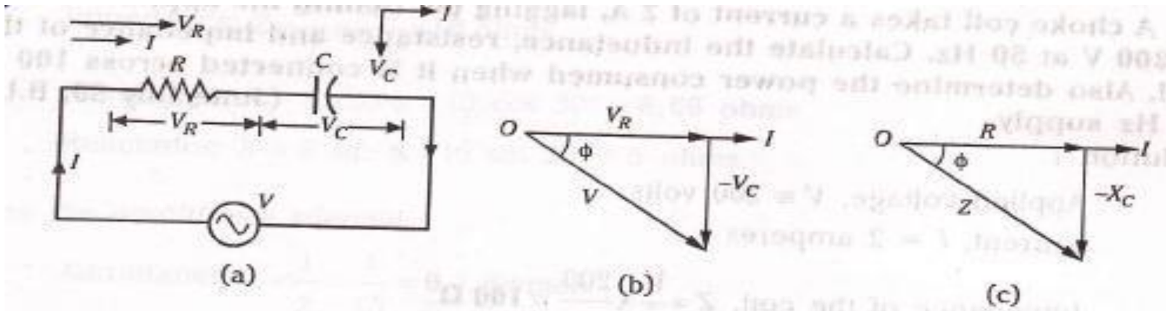


Fig. 3.52

Consider an a.c. circuit containing resistance R ohms and capacitance C farads, as shown in the fig. 3.52(a).

Let V = r.m.s. value of voltage
I = r.m.s. value of current

$\therefore$ voltage drop across R, $V_R = IR$ - in phase with I

Voltage drop across C, $V_C = IX_C$ - lagging I by $\frac{\pi}{2}$

The capacitive resistance is negative, so V_C is in the negative direction of Y – axis, as shown in the fig. 3.52(b).

We have $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2}$

$= \sqrt{R^2 + X_C{}^2}$

Or $I = \frac{V}{\sqrt{R^2 + X_C{}^2}} = \frac{V}{Z}$

The denominator, Z is the impedance of the circuit, i.e., $Z = \sqrt{R^2 + X_C{}^2}$. fig. 3.52(c) depicts the impedance triangle.

Power factor, $\cos\phi = \frac{R}{Z}$

Fig. 3.52(b) shows that I leads V by an

angle ϕ, so that $\tan\phi = \frac{-X_C}{R}$

This implies that if the alternating voltage is $v = V_m \sin \omega t$, the resultant current in the R – C circuit is given by

$i = I_m \sin(\omega t + \phi)$, such that current leads the applied voltage by the angle ϕ.

fig. 3.53 depict this.

The waveforms of

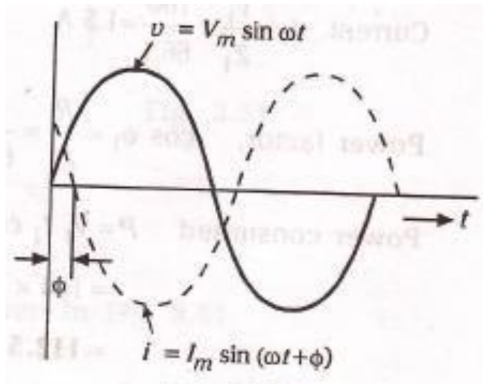


Fig. 3.53

Power: Average power, P = v × I = VI cos ϕ (as in sec. 3.17).

Power curves: The power curve for R – C series circuit is shown in fig. 3.54. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power is positive.

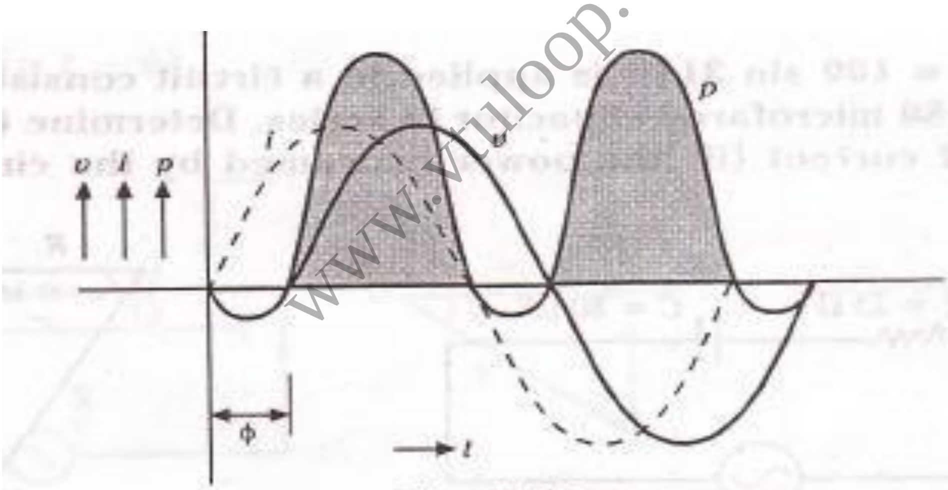


Fig. 3.54

Resistance, Inductance and capacitance in series (RLC – Series Circuit)

Consider an a.c. series circuit containing resistance R ohms, Inductance L henries and capacitance C farads, as shown in the fig. 3.59.

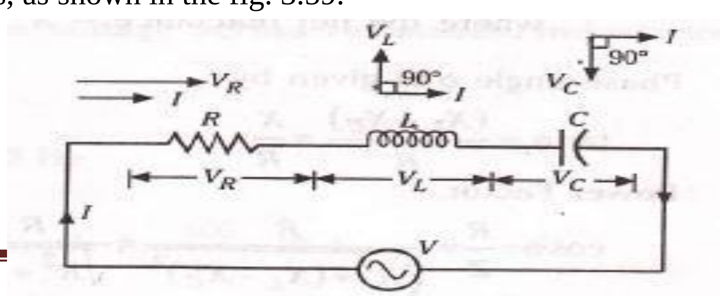


Fig. 3.59

Let V = r.m.s. value of applied voltage

I = r.m.s. value of current

∴ Voltage drop across R, VR = IR - in phase with I

voltage drop across L, VL = $I.X_L$ - lagging I by 90^0

Voltage drop across C, $V_C = IX_C$ - lagging I by 90^0

Referring to the voltage triangle of Fig. 3.60, OA represents V_R, AB and AC represent inductive and capacitive drops respectively. We observe that V_L and V_C are 180^0 out of phase.

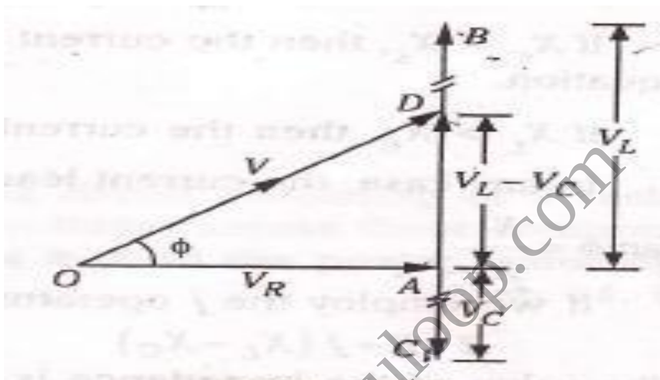


Fig. 3.60

Thus, the net reactive drop across the combination is

AD = AB – AC

= AB – BD (∵ BD = AC)

= $V_L – V_C$

= I (X_L - X_C)

OD, which represents the applied voltage V, is the vector sum of OA and AD.

∴ $OD = \sqrt{OA^2 + AD^2}$ OR $V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$= I\sqrt{R^2 + (X_L - X_C)^2}$

Or $I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{V}{\sqrt{R^2+X^2}} = \frac{V}{Z}$

The denominator $\sqrt{R^2 + (X_L - X_C)^2}$ is the impendence of the circuit.

So (impedance)^2 = (resistance)^2 + (net reactance)^2

Or $Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$

Where the net reactance = X (fig. 3.61)

Phase angle ϕ is given by

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{X}{R}$$

power factor,

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

Power = VI cos ϕ

If applied voltage is represented by the equation $v = V_m \sin\omega t$, then the resulting current in an R – L – C circuit is given by the equation

$$i = I_m \sin(\omega t \pm \phi)$$

If $X_C > X_L$, then the current leads and the +ve sign is to be used in the above equation.

If $X_L > X_C$, then the current lags and the –ve sign is to be used.

If any case, the current leads or lags the supply voltage by an angle ϕ, so that $\tan\phi = \frac{X}{R}$

If we employ the j operator (fig. 3.62), then we have

$$Z = R + j\,(X_L - X_C)$$

The value of the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle $\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$

$$Z\angle\phi = Z\angle \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

$$= Z\angle \tan^{-}\left[\frac{X}{R}\right]$$

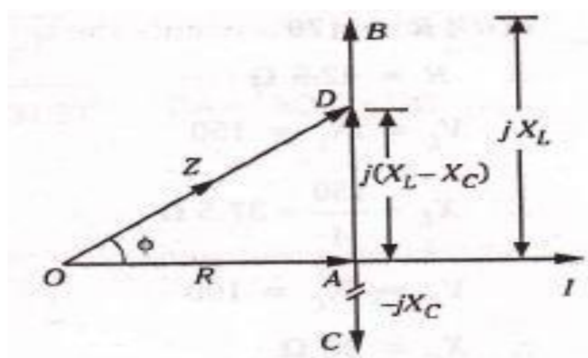


Fig. 3.62

Three Phase Circuits:

Advantages of three phase system:

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 1200 between each other. Such a three phase system has following advantages over single phase system:

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.
3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self-starting.
4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5) Three phase system give steady output.
6) Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.
7) Power factor of single phase motors is poor than three phase motors of same rating.
8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard polyphase system throughout the world.

Generation of 3-phase E.M.F.

In the 3-phase system, there are three equal voltages of the same frequency but displaced from one another by 120^0 electrical. These voltages are produced by a three-phase generator which has three identical windings or phases displaced 120^0 electrical apart. When these windings are rotated in a magnetic field, e.m.f. is induced in each winding or phase. These e.m.f. s are of the same magnitude and frequency but are displaced from one another by 120^0 electrical.

Consider three electrical coils a_1a_2, b_1b_2 and c_1c_2 mounted on the same axis but displaced from each other by 120^0 electrical. Let the three coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of ω radians/sec, as shown in Fig. 3.80. Here, a_1, b_1 and c_1 are the start terminals and a_2, b_2 and c_2 are the end terminals of the coils.

When the coil a_1a_2 is in the position AB shown in Fig. 3.80, the magnitude and direction of the e.m.f. s induced in the various coils is as under:

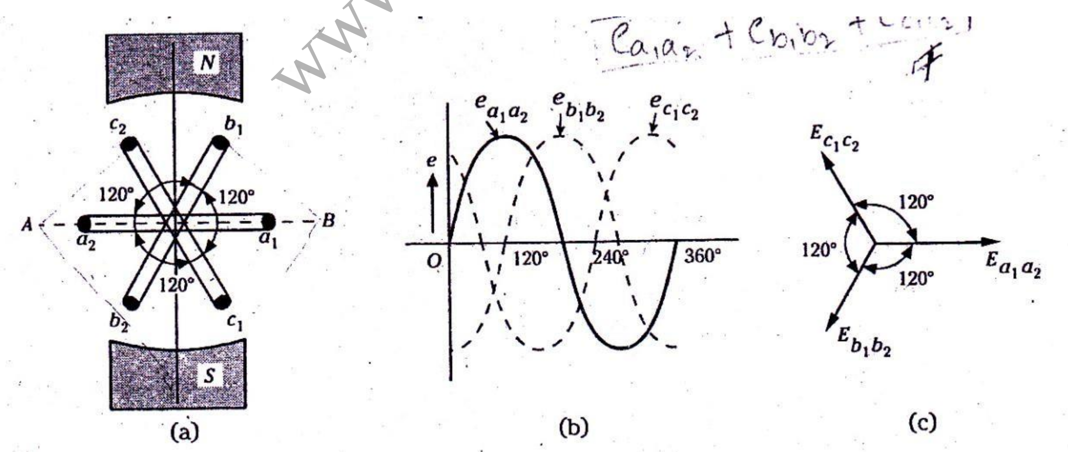


Fig. 3.80

a) E.m.f. induced in coil $a_1 a_2$ is zero and is increasing in the positive direction. This is indicated by $e_{a1\,a2}$ wave in Fig. 3.80 (b).

b) The coil $b_1 b_2$ is 120^0 electrically behind coil $a_1 a_2$. the e.m.f. induced in this coil is negative and is approaching maximum negative value. This is shown by the $e_{b1\,b2}$ wave.

c) The coil $c_1 c_2$ is 240^0 electrically behind $a_1 a_2$ or 120^0 electrically behind coil $b_1 b_2$. The e.m.f. induced in this coil is positive and is decreasing. This is indicated by wave $e_{c1\,c2}$. Thus, it is apparent that the e.m.f.‟s induced in the three coils are of the same magnitude and frequency but displaced 120^0 electrical from each other.

Vector Diagram: The r.m.s. values of the three phase voltage are shown vectorially in Fig. 3.80(c).

Equations: The equations for the three voltages are:

$$e_{a1\,a2} = E_m \sin \omega t$$

$$e_{b1\,b2} = E_m \sin \omega t \quad ; \quad e_{c1\,c2} = E_m \sin \omega t$$

Meaning of phase sequence

The order in which the voltages in the voltages in the phases reach their maximum positive values is called the phase sequence. For example, in Fig. 3.80(a), the three coils $a_1 a_2$, $b_1 b_2$ and $c_1 c_2$ are rotating in anticlockwise direction in the magnetic field. The coil $a_1 a_2$ is 120^0 electrical ahead of coil $b_1 b_2$ and 240^0 electrical ahead of coil $c_1 c_2$. Therefore, e.m.f. in coil $a_1 a_2$ leads the e.m.f. in coil $b_2 b_2$ by 120^0 and that in coil $c_1 c_2$ by 240^0. It is evident from Fig. 3.80(b) that $e_{a1\,a2}$ attains maximum positive first, then $e_{b1\,b2}$ and $e_{c1\,c2}$. In other words, the order in which the e.m.f. s in the three phases $a_1 a_2$, $b_1 b_2$ and $c_1 c_2$ attain their maximum positive values is a,b,c. Hence, the phase sequence is a,b,c.

Naming the phases

The 3 phases may be numbered (1,2,3) or lettered (a,b,c) or specified colours (R Y B). By normal convention, sequence RYB is considered positive and R B Y negative.

Meaning of phase sequence

It is necessary to employ some systematic notation for the solution of a.c. circuits and systems containing a number of e.m.f. s. acting and currents flowing so that the process of solution is simplified and less prone to errors.

It is normally preferred to employ double-subscript notation while dealing with a.c. electrical circuits. In this system, the order in which the subscripts are written indicates the direction in which e.m.f. acts or current flows.

For example, if e.m.f. is expressed as E_{ab}, it indicates that e.m.f. acts from a to b; if it is expressed as E_{ba}, then the e.m.f. acts in a direction opposite to that in which E_{ab} acts. (Fig. 3.81) i.e., $E_{ba} = - E_{ab}$.

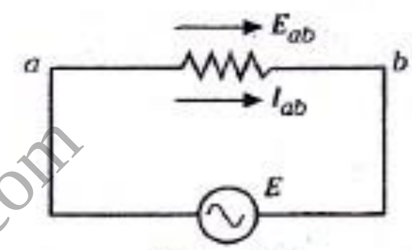


Fig. 3.81

Similarly, I_{ab} indicates that current flows in the direction from a to b but I_{ba} indicates that current flows in the direction from b to a; i.e., $I_{ba} = -I_{ab}$.

Balanced Supply and Load

When a balanced generating supply, where the three phase voltages are equal, and the phase difference is 120^0 between one another, supplies balanced equipment load, where the impedance of the three phases or three circuit loads are equal, then the current flowing through these three phases will also be equal in magnitude, and will also have a phase difference of 120^0 with one another. Such an arrangement is called a balanced load.

Obtaining Relationship between Line & Phase Values & Expression for power for Balanced Star Connection

This system is obtained by joining together similar ends, either the start or the finish; the other ends are joined to the line wires, as shown in Fig.3.82 (a). The common point N at which similar (start or finish) ends are connected is called the neutral or star point. Normally, only three wires are carried to the external circuit, giving a 3-phase, 3-wire, star-connected system; however, sometimes a fourth wire known as neutral wire, is carried to the neutral point of the external load circuit, giving a 3-phase, 4-wire connected system.

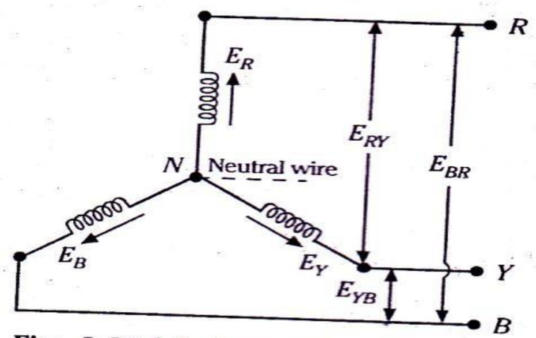


Fig. 3.82 (a) Connection Diagram

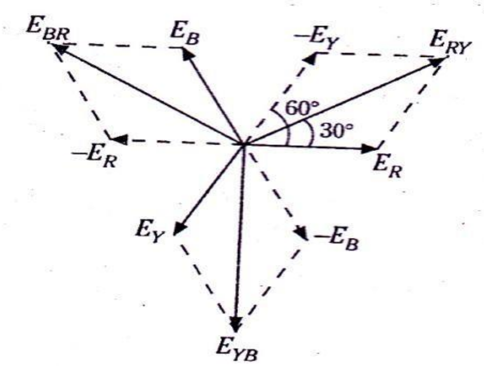


Fig. 3.82 (b) Vector Diagram of Line and Phase voltages. 3-Phase Star-Connected System

The voltage between any line and the neutral point, i.e., voltage across the phase winding, is called the phase voltage; while the voltage between any two outers is called line voltage. Usually, the neutral point is connected to earth. In Fig.3.82 (a), positive directions of e.m.f.s. are taken star point outwards. The arrow heads on e.m.f.s. and currents indicate the positive direction. Here, the 3-phases are numbered as usual: R,Y and B indicate the three natural colours red, yellow and blue respectively. By convention, sequence RYB is taken as positive and RYB as negative.

In Fig.3.82 (b), the e.m.f.s induced in the three phases, are shown vectorially. In a star-connection there are two windings between each pair of outers and due to joining of similar ends together, the e.m.f.s induced in them are in opposition.

Hence the potential difference between the two outers, know as line voltage, is the vector difference of phase e.m.f.s of the two phases concerned.

For example, the potential difference between outers R and Y or

Line voltage E_{RY}, is the vector difference of phase e.m.f.s E_R and E_Y or vector sum of phase e.m.f.s E_R and ($-E_Y$).

i.e. $E_{RY} = E_R - E_Y$ (vector difference)

or $E_{RY} = E_R + (-E_Y)$ (vector sum)

as phase angle between vectors E_R and $(-E_Y)$ is 600,

$\therefore$ from vector diagram shown in Fig.3.82(b),

$$E_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^0}$$

or $E_{RY} = E_R + (-E_Y)$ (vector sum)

as phase angle between vectors E_R and $(-E_Y)$ is 600,

$\therefore$ from vector diagram shown in Fig.3.82(b),

$$E_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^0}$$

Let $E_R = E_Y = E_B = E_P$ (phase voltage)

Then line voltage $E_{Ry} = \sqrt{E_P^2 + E_P^2 + (2E_P E_P \times 0.5)} = \sqrt{3}\, E_P$

Similarly, potential difference between outers Y and B or line. Voltage $E_{YB} = E_Y - E_B = \sqrt{3}\, E_P$ and potential difference between outers B and R, or line voltage $E_{BR} = E_B - E_R = \sqrt{3}\, E_P$.

In a balanced star system, E_{RY}, E_{YB} and E_{BR} are equal in magnitude and are called line voltages.

$$\therefore \quad E_L = \sqrt{3}\, E_P$$

Since, in a star-connected system, each line conductor is connected to a separate phase, so the current flowing through the lines and phases are the same.

i.e. Line current I_L = phase current I_P

If the phase current has a phase difference of ϕ with the voltage,

Power output per phase = $E_P I_P \cos \phi$

Total power output, $P = 3E_P I_P \cos\phi$

$$= 3\frac{E_L}{\sqrt{3}} I_P \cos\phi$$

$$= \sqrt{3}\, E_L\, I_L \cos\phi$$

i.e. power = $\sqrt{3}$ x line voltage x line current x power factor

Apparent power of 3-phase star-connected system

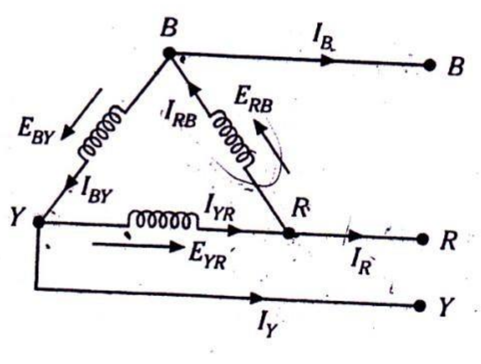
= 3 x apparent power per phase

$$= 3E_P I_P = 3\text{x}\,\frac{E_L}{\sqrt{3}}\,\text{x}\,I_L = \sqrt{3}\,E_L\,I_L$$

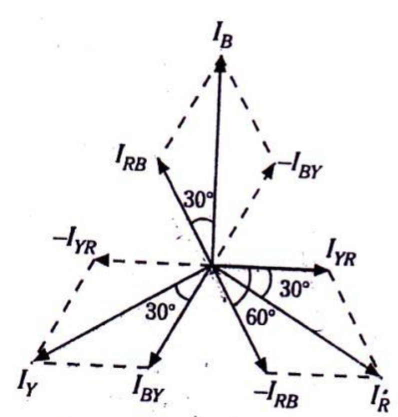
Obtaining Relationship between Line and Phase Values and Expression for Power for Balanced Delta Connection

When the starting end of one coil is connection to the finishing end of another coil, as shown in Fig.3.83 (a), delta or mesh connection is obtained. The direction of the e.m.f.s is as shown in the diagram.

From Fig.3.83 it is clear that line current is the vector difference of phase currents of the two phases concerned. For example, the line current in red outer I_R will be equal to the vector difference of phase currents I_{YR} and IRB. The current vectors are shown in Fig.3.83 (b).



(a) Connection Diagram



(b) Vector Diagram

Fig. 3.83

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Referring to Fig.3.83 (a) and (b),

Line current, $I_R = I_{YR} - I_{RB}$ (vector difference)

$= I_{YR} + (-I_{RB})$ (vector sum)

As the phase angle between currents I_{YR} and $-I_{RB}$ is 600

$$\therefore \quad I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_{YR}I_{RB}\cos 60^0}$$

For a balanced load, the phase current in each winding is equal and let it be = I_P

$$\therefore \text{ Line current, } I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_P I_P x0.5} = \sqrt{3}\, I_P$$

Similarly, line current, $I_Y = I_{BY} - I_{YR} = \sqrt{3}\, I_P$

And line current, $I_B = I_{RB} - I_{BY} = \sqrt{3}\, I_P$

In a delta network, there is only one phase between any pair of line outers, so the potential difference between the outers, called the line voltage, is equal to phase voltage.

i.e. Line voltage, E_L = phase voltage, E_P

Power output per phase = $E_P I_P \cos\phi$,; where $\cos\phi$ is the power factor of the load.

Total power output, $P = 3E_P I_P \cos\phi$

$$= 3E_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3}\, E_L I_L \cos\phi$$

i.e. Total power output = $\sqrt{3}$ x Line voltage x Line current x p.f.

Apparent power of 3-phase delta-connected system

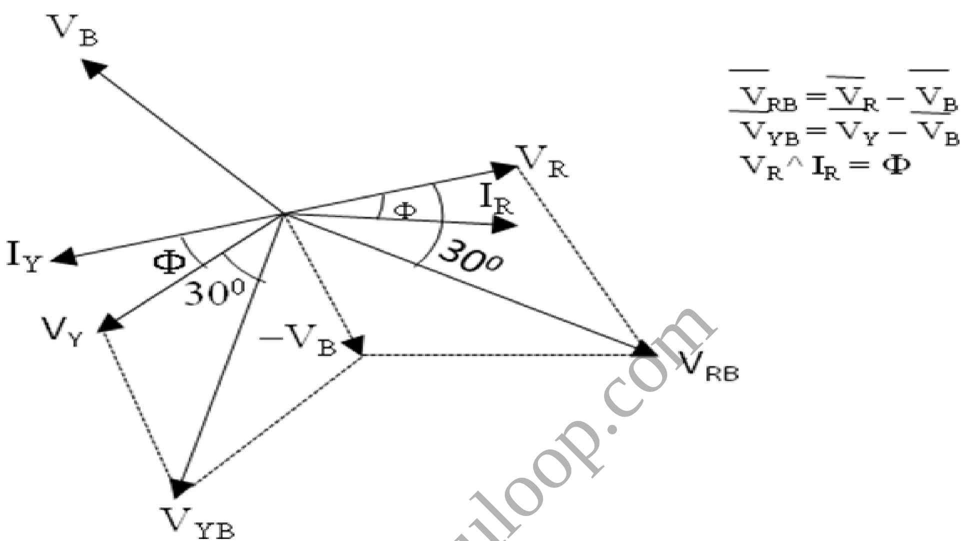
= 3 x apparent power per phase

$$= 3\, E_P I_P = 3E_L \frac{I_L}{\sqrt{3}} = \sqrt{3}\, E_L I_L$$

Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the

total three phase power and power factor of the circuit.

Two wattmeter method: The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeters is connected between its own current coil terminal and line without current coil. Consider star connected balanced load and two wattmeters connected as shown in fig. 13. Let us consider the rms values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.



$V_{RB} = V_R - V_B$; $V_{YB} = V_Y - V_B$; $V_R \wedge I_R = \Phi$

$V_R = V_Y = V_B = V_{ph}$

$V_{RB} = V_R - V_B$, $I_R = I_Y = I_L = I_{ph}$

$V_{YB} = V_Y - V_B = V_{RB} = V_L$

From fig. 14, $I_R \wedge V_{RB} = 30 - \Phi$ and $I_R \wedge V_{RB} = 30 + \Phi$

$$W_1 = I_R V_{RB} \cos(30 - \Phi) = V_L I_L \cos(30 - \Phi)$$

$$W_2 = I_Y V_{YB} \cos(30 + \Phi) = V_L I_L \cos(30 + \Phi)$$

$$W_1 + W_2 = V_L I_L[\cos(30 - \Phi) + \cos(30 + \Phi)]$$
$$= V_L I_L[\cos 30 \cos \Phi + \sin 30 \sin \Phi + \cos 30 \cos \Phi - \sin 30 \sin \Phi]$$
$$= 2V_L I_L \cos 30 \cos \Phi = 2V_L I_L \frac{\sqrt{3}}{2} \cos \Phi$$