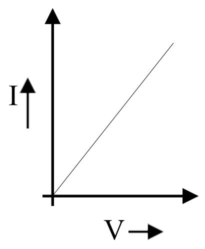
MODULE - 1

Ohm's law : " The potential difference between the two ends of a conductor is directly proportional to the current flowing through it, provided its temperature & other parameters remain unchanged ".



$$V \propto I$$

$$V = R\,I$$

R - Constant of proportionality called Resistance
V - voltage in volts
I - current in Amps
R - resistance in Ohms (Ω)

<u>Limitations of Ohm's law :</u>

- It is not applicable to <u>non-metallic conductors</u> like silicon carbide. Their v-i relationship is given by ,

  [$V = K I^m$] - Here the relation between V & I is <u>non linear</u>. ($m < 1$)

- It is not applicable to <u>non-linear devices</u> like <u>diodes</u>.
- It is not applicable to <u>'arc lamps'</u>, because arc produced exhibits non-linear characteristics.

Kirchhoff's laws :-

I. <u>Kirchhoff's current law</u> [KCL] -

" Algebraic sum of all currents meeting at a node is zero in any electric circuit"

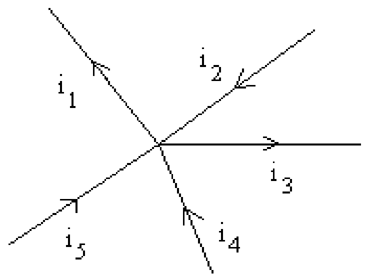
OR

" Sum of all currents entering a node is equal to sum of all currents leaving a node in any electrical circuit."

KCL is illustrated below .

Convention

Current flowing **out** of a node is considered positive (+ve)
Current flowing **into** a node is considered negative (- ve)



$$i_1 - i_2 + i_3 - i_4 - i_5 = 0$$

or

$$i_1 + i_3 = i_2 + i_4 + i_5$$

II. Kirchhoff's voltage law [KVL] -

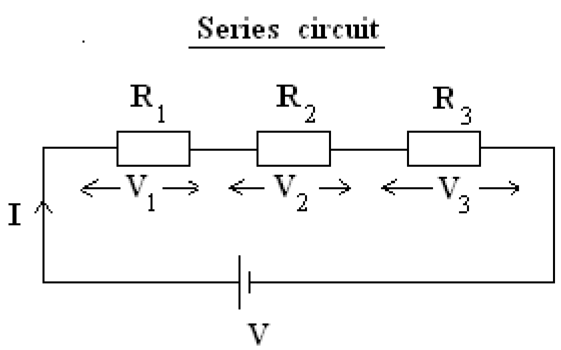
" In any electrical circuit, the algebraic sum of voltage drops of all branches & emf's is zero."

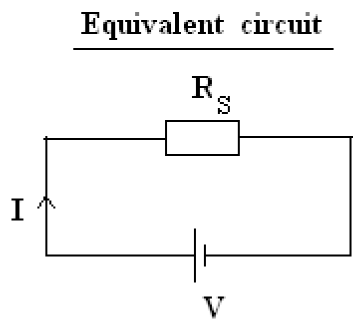
or

" In any electrical network, the algebraic sum of voltage drops of all branches and emf's forming a closed loop is zero."

Series , parallel , series-parallel combination circuits:-

Resistances in series :-



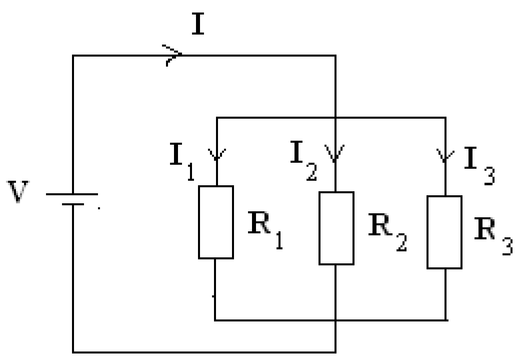


$$V = V_1 + V_2 + V_3$$

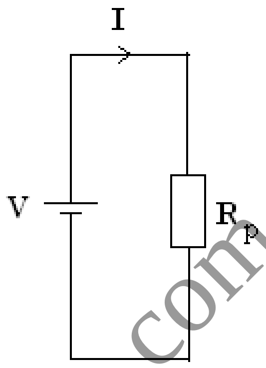
$$IR_s = IR_1 + IR_2 + IR_3$$

$$\boxed{R_s = R_1 + R_2 + R_3}$$

Resistances in parallel :-



Equivalent circuit



$$I = I_1 + I_2 + I_3$$

$$V/R_p = V/R_1 + V/R_2 + V/R_3$$

$$\boxed{1/R_p = 1/R_1 + 1/R_2 + 1/R_3}$$

1. Find the values of currents in all the branches of the network shown in figure

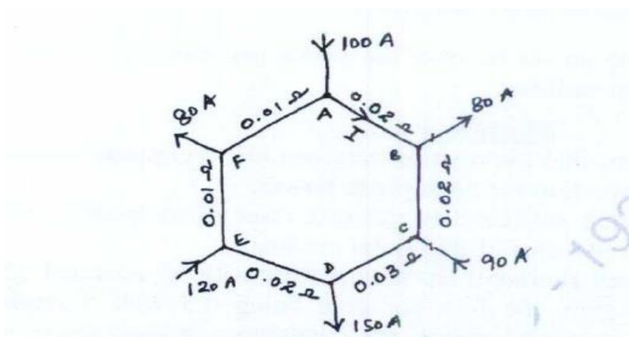


Fig. Q1 (b)

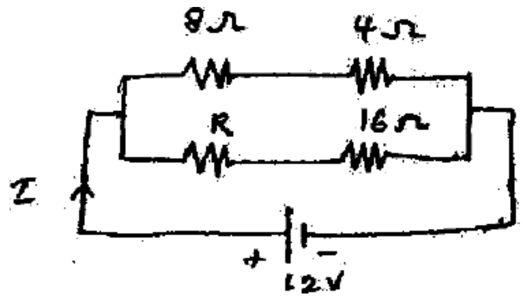
$$-0.2I-0.1(I-60)-0.3I-0.1(I-120)-0.1(I-50)-0.2(I-80) = 0$$

$$-0.2I-0.1I-0.3I-0.1I-0.1I-0.2I+6+12+5+16 = 0$$

$$-I = -39$$

$$I = 39$$

2. If the total power dissipated in the circuit shown is 18W, find the value of ‘R’ and its current.



P = 18W

I = P/V = 18/12 = 1.5 A

I_1 = V/ (8+4) (Since it is a parallel circuit).

= 12/12 = 1A

$I_2 = I - I_1$

= 1.5 – 1 = 0.5A

Voltage across 16Ω resistor is $V_{16\Omega} = I_2 \times 16 = 0.5 \times 16 = 8V$

So voltage Across R is 12- 8 = 4 V

R = V/ I_2 = 4/0.5 = 8Ω

3. A current of 20A flows through two ammeters A and B in series. The potential difference across A is 0.2V and across B is 0.3V.Find how the same current will divide between A and B when they are in parallel .

Case I:

V1 = 0.2 V

V2 = 0.3 V

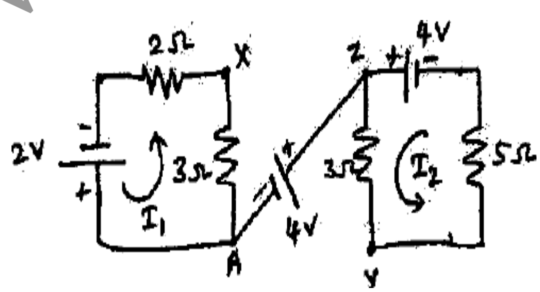
Resistance of ammeter A, R1 = V1/I = 0.2/20 = 0.01 ohms

Resistance of ammeter B, R2 = V2/I = 0.3/20 = 0.015 ohms

Case II:

I1 = I* (R2/ (R1+R2)) = 20*(0.015/ (0.015+0.01)) = 12A I2 = I* (R1/ (R1+R2)) = 20*(0.01/ (0.015+0.01)) = 8A

4. What is the potential difference between the point x and y in the network shown ?



$I_1 = 2/(2+3) = 0.4A$

$I_2 = 4/(3+5) = 0.5A$

$V_{xy} = 3I_1 + 4 - 3I_2$

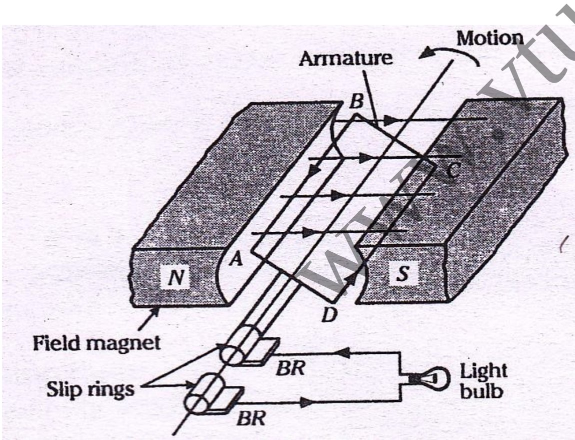
$= (3*0.4) + 4 - (3*0.5)$

$= 3.7\ V$

Generation of sinusoidal AC Voltage:

Alternating voltage may be generated:

a) By rotating a coil in a magnetic field as shown in Fig.3.1.
b) By rotating a magnetic field within a stationary coil as shown in Fig.3.2.



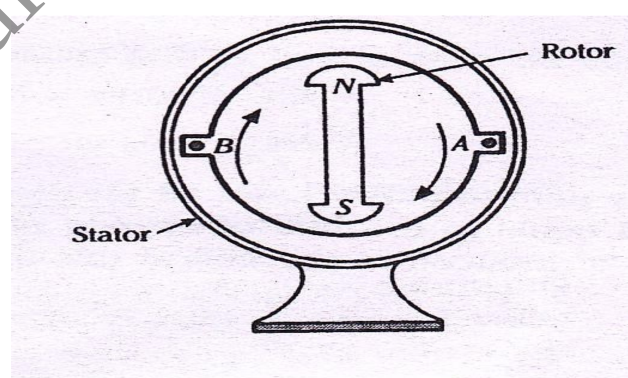


Fig. 3.2

" In each case, the value of the alternating voltage generated depends upon the number of turns in the coil, the strength of the field and the speed at which the coil or magnetic field rotates."

The alternating voltage generated has regular changes in magnitude and direction. If a load resistance (e.g. a light bulb) is connected across this alternating voltage, an alternating current flows in the circuit. When there is a reversal of polarity of the alternating voltage, the direction of current flow in the circuit also reverses.

Equation of Alternating E.M.F.

Let us take up the case of a rectangular coil of N turns rotating in the anticlockwise direction, with an angular velocity of ω radians per second in a uniform magnetic field as shown in Fig.3.3. let the time be measured from the instant of coincidence of the plane of the coil with the

X-axis. At this instant maximum flux ϕ_{max}" links with the coil. As the coil rotates, the flux linking with it changes and hence e.m.f. is induced in it. Let the coil turn through an angle θ in time„t" seconds, and let it assume the position as shown in Fig.3.3. Obviously θ = ωt.

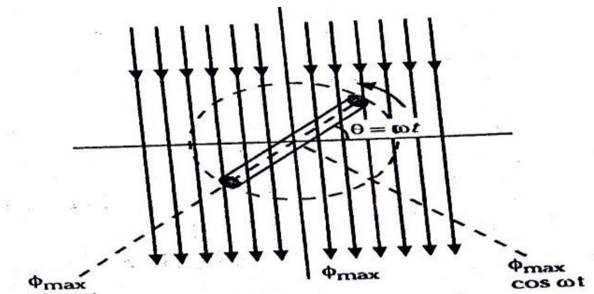


Fig. 3.3

When the coil is in this position, the maximum flux acting vertically downwards can be resolved into two components, each perpendicular to the other, namely:

a) Component ϕ_{max} sin ωt, parallel to the plane of the coil. This component does not induce e.m.f. as it is parallel to the plane of the coil.
b) Component ϕ_{max} cos ωt, perpendicular to the plane of coil. This component induces e.m.f. in the coil.

∴ flux linkages of coil at that instant (at θ^0) is

= No. of turns x flux linking

$= N \phi_{max} \cos \omega t$

As per faraday"s Laws of Electromagnetic induction, the e.m.f. induced in a coil is equal to the rate of change of flux linkages of the coil. So, instantaneous e.m.f. „e" induced in the coil at this instant is:

$$e = -\frac{d}{dt} \text{(flux linkages)}$$

$$= -\frac{d}{dt}(N\phi_{max} \cos \omega t)$$

$$= -N\phi_{max} \frac{d}{dt}(\cos \omega t)$$

$$= -N\phi_{max}\,\omega(-\sin \omega t)$$

$$\therefore \quad e = +N\phi_{max} \sin \omega t \text{ volts} \qquad \ldots (1)$$

It is apparent from eqn.(1) that the value of „e" will be maximum (E_m), when the coil has rotated through 90^0 (as $\sin 90^0 = 1$)

$$\text{Thus} \quad E_m = N\,\omega\,\phi_{max} \text{ volts} \qquad \ldots (2)$$

Substituting the value of $N \omega \phi_{max}$ from eqn.(2) in eqn.(1), we obtain:

$$e = E_m \sin \omega t \qquad . . .(3)$$

We know that $\theta = \omega t$

$$\therefore\ e = E_m \sin \theta$$

It is clear from this expression of alternating e.m.f. induced in the coil that instantaneous e.m.f. varies as the sin of the time angle (θ or ωt).

$\omega = 2\pi f$, where „f" is the frequency of rotation of the coil. Hence eqn.(3) can be written

$$\text{as } e = E_m \sin 2\pi ft \qquad . . . (4)$$

If T = time of the alternating voltage = $\frac{1}{f}$, then eqn.(iv) may be re-written as

$$e = E_m \sin\left(\frac{2\pi}{T}\right)t$$

so, the e.m.f. induced varies as the sine function of the time angle, ωt, and if e.m.f. induced is plotted against time, a curve of sine wave shape is obtained as shown in Fig.3.4. Such an e.m.f. is called sinusoidal when the coil moves through an angle of 2π radians.

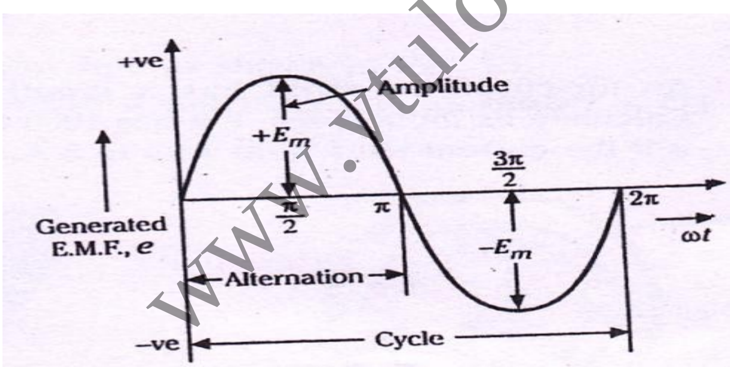


Fig. 3.4

Equation of Alternating Current

When an alternating voltage $e = E_m \sin \omega t$ is applied across a load, alternating current flows through the circuit which will also have a sinusoidal variation. The expression for the alternating current is given by:

$$i = I_m \sin \omega t$$

In this case the load is resistive (we shall see, later on, that if the load is inductive or capacitive, this current-equation is changed in time angle).

Important Definitions

Important terms/definitions, which are frequently used while dealing with a.c. circuits, are as given below:

1. Alternating quantity: An alternating quantity is one which acts in alternate positive and negative directions, whose magnitude undergoes a definite series of changes in definite intervals of time and in which the sequence of changes while negative is identical with the sequence of changes while positive.
2. Waveform: "The graph between an alternating quantity (voltage or current) and time is called waveform", generally, alternating quantity is depicted along the Y-axis and time along the X-axis.fig.4.4 shows the waveform of a sinusoidal voltage.
3. Instantaneous value: The value of an alternating quantity at any instant is called instantaneous value.
   The instantaneous values of alternating voltages and current are represented by „e" and „I" respectively.
4. Alternation and cycle: When an alternating quantity goes through one half cycle (complete set of +ve or –ve values) it completes an alternation, and when it goes through a complete set of +ve and –ve values, it is said to have completed one cycle.
5. Periodic Time and Frequency: The time taken in seconds by an alternating quantity to complete one cycle is known as periodic time and is denoted by T.
   The number of cycles completed per second by an alternating quantity is know as frequency and is denoted by „f". in the SI system, the frequency is expressed in hertz.

   The number of cycles completed per second = f.

   Periodic Time T – Time taken in completing one cycle = $\frac{1}{f}$

   Or $f = \frac{1}{T}$

   In India, the standard frequency for power supply is 50 Hz. It means that alternating voltage or current completes 50 cycles in one second.

6. Amplitude: The maximum value, positive or negative, which an alternating quantity attains during one complete cycle, is called amplitude or peak value or maximum value. The amplitude of alternating voltage and current is represented by E_m and I_m respectively.

Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already mentioned in sec.3.2 is

$$e = Em \sin \theta = Em \sin \omega t = Em \sin 2\pi f\, t = Em \sin \frac{2\pi}{T} t$$

on perusal of the above equations, we find that

a) The amplitude or peak value or maximum value of an alternating voltage is given by the coefficient of the sine of the time angle.
b) The frequency „f" is given by the coefficient of time divided by 2π.

Taking an example, if the equation is of an alternating voltages is given by e = 20 sin 314t, then its maximum value is 20 V and its frequency is

$$f = \frac{314}{2\pi}\ 50 \text{ Hz}$$

In a like manner, if the equation is of the form

$$e = I_m\sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\,\omega t$$, then its maximum value is

$$E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$$ and the frequency is

$$\frac{2\omega}{2\pi} \text{ Or } \frac{\omega}{\pi} \text{ Hertz}$$

Root-mean-square (R.M.S.) Value:

The r.m.s. or effective value, of an alternating current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

Let us take two circuits with identical resistance, but one is connected to a battery and the other to a sinusoidal voltage source. Wattmeters are employed to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that the heat power produced in each circuit is the same. In this event the direct current I will equal $\frac{I_m}{\sqrt{2}}$, which is termed r.m.s. value of the sinusoidal current.

The following method is used for finding the r.m.s. or effective value of sinusoidal waves.

The equation of an alternating current varying sinusoid ally is given by i= I_m sin θ.

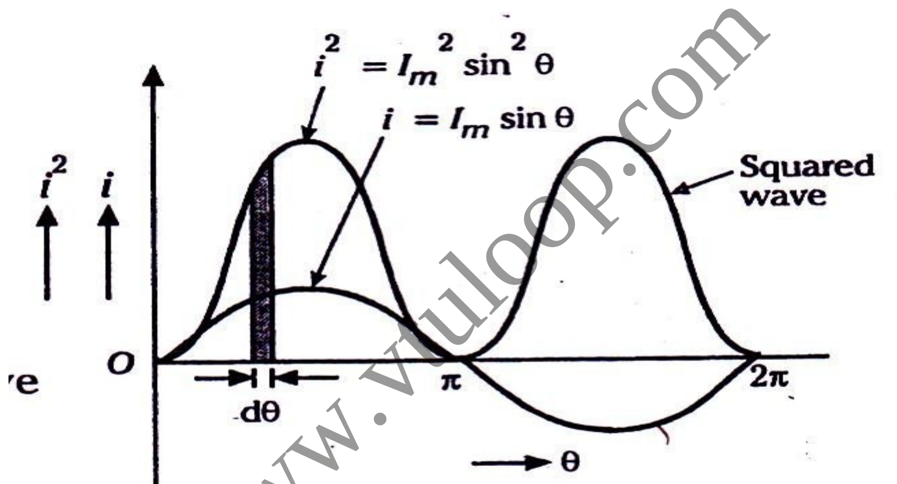


Fig. 3.5

Let us consider an elementary strip of thickness dθ in the first cycle of the squared wave, as shown in Fig.3.5.

Let i^2 be mid-ordinate of this strip.

Area of the strip = i^2 d θ

Area of first half-cycle of squared wave

$$= \int_0^{\pi} i^2 \, d\theta$$

$$= \int_0^{\pi} (I_m \sin\theta)^2 \, d\theta \qquad (\because I = I_m \sin\theta)$$

$$= \int_0^{\pi} I_m{}^2 \sin^2\theta \, d\theta$$

$= I_m^2 \int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$ $\quad (\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \frac{I_m^2}{2} \int_0^{\pi} (1-\cos 2\theta) d\theta$

$= \frac{I_m^2}{2} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$

$= \frac{I_m^2}{2} \left[(\pi - 0) - (0 - 0)\right]$

$= \frac{\pi I_m^2}{2}$

$\therefore I = \sqrt{\frac{\text{Area of first half cycle of squared wave}}{\text{base}}}$

$= \sqrt{\frac{\pi I_m^2}{2} \times \frac{1}{\pi}}$

$= \sqrt{\frac{I_m^2}{2}}$

$= \frac{I_m}{\sqrt{2}} = 0.707 I_m$

Hence, for a sinusoidal current,

R.M.S. value of current = 0.707 x maximum value of current.

Similarly, $E = 0.707\ E_m$

Average Value

The arithmetical average of all the values of an alternating quantity over one cycle is called **average value.**

In the case of a symmetrical wave e.g. sinusoidal current or voltage wave, the positive half is exactly equal to the negative half, so that the average value over the entire cycle is zero. Hence, in this case, the average value is obtained by adding or integrating the instantaneous values of current over one alternation (half-cycle) only.

The equation of a sinusoidally varying voltage

Is given by $e = E_m \sin \theta$.

Let us take an elementary strip of thickness dθ in the first half-cycle as shown in Fig.3.6. let the mid-ordinate of this strip be „e‟.

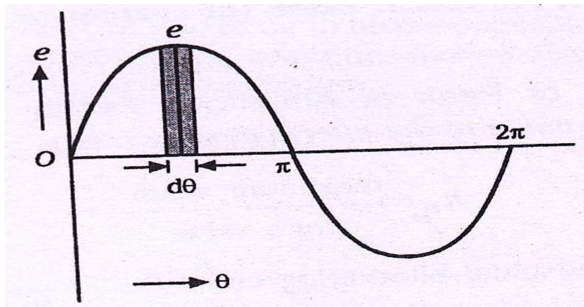


Fig. 3.6

Area of the strip = e dθ

Area of first half-cycle

$= \int_0^{\pi} e \, d\theta$

$= \int_0^{\pi} E_m \sin\theta \, d\theta$ (∵ $e = E_m \sin\theta$)

$= E_m \int_0^{\pi} \sin\theta \, d\theta$

$= E_m \left[-\cos\theta\right]_0^{\pi} = 2E_m$

∴ Average value, $E_{av} = \frac{\text{Area of half cycle}}{\text{base}} = \frac{2E_m}{\pi}$

Or E_{av} = 0.637 Em

In a similar manner, we can prove that, for alternating current varying sinusoidally,

$I_{av} = 0.637\ I_m$

∴ **Average value of current = 0.637 x maximum value**

Form Factor and crest or peak or Amplitude Factor (K_f)

A definite relationship exists between crest value (or peak value), average value and r.m.s. value of an alternating quantity.

1. Form Factor: The ratio of effective value (or r.m.s. value) to average value of an alternating quantity (voltage or current) is called form factor, i.e.

From Factor, $K_f = \frac{\text{rm s value}}{\text{average value}}$

For sinusoidal alternating current,

$$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

For sinusoidal alternating voltage,

$$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

Hence, the R.M.S. value (of current or voltage) is 1.11 times its average value.

2. Crest or Peak or Amplitude Factor (Ka): It is defined as the ratio of maximum value to the effective value (r.m.s. value) of an alternating quantity. i.e.,

$$Ka = \frac{\text{maximum value}}{\text{r.m.s.value}}$$

For sinusoidal alternating current,

$$Ka = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

For sinusoidal alternating voltage,

$$Ka = \frac{E_m}{\frac{E_m}{\sqrt{2}}} = 1.414$$

The knowledge of Crest Factor is particularly important in the testing of dielectric strength of insulating materials; this is because the breakdown of insulating materials depends upon the maximum value of voltage.

Phase

An alternating voltage or current changes in magnitude and direction at every instant. So, it is necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any particular instant is called its phase.

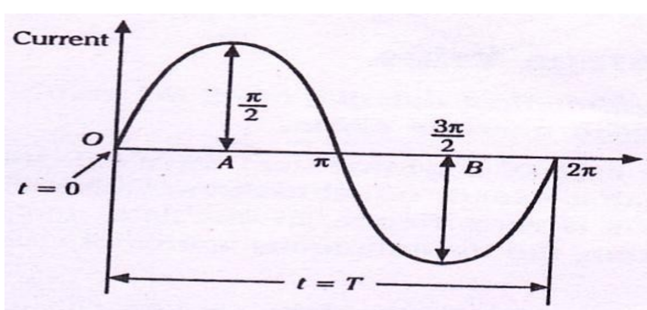


Fig. 3.7

We may define the phase of an alternating quantity at any particular instant as the fractional part of a period or cycle through which the quantity has advanced from the selected origin.

Taking an example, the phase of current at point A (+ve maximum value) is T/4 second, where T is the time period, or expressed in terms of angle, it is π/2 radians (Fig.3.7). In other words, it means that the condition of the wave, after having advanced through π/2 radians (900) from the selected origin (i.e.,0) is that it is maximum value (in the positive direction).similarly, -ve maximum value is reached after 3π/2 radians (2700) from the origin, and the phase of the current at point B is 3T/4 second.

Phase Difference (Lagging or Leading of Sinusoidal wave)

When two alternating quantities, say, two voltages or two currents or one voltage and one current are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant.

One may pass through its maximum value at the instant when the other passes through a value other than its maximum one. These two quantities are said to have a phase difference. Phase difference is specified either in degrees or in radians.

The phase difference is measured by the angular difference between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to lead the other quantity, whereas the second quantity is said to lag behind the first one. In Fig.3.8, current I_1, represented by vector 0A, leads the current I_2, represented by vector 0B, by ϕ, or current I_2 lags behind the current I_1 by ϕ.

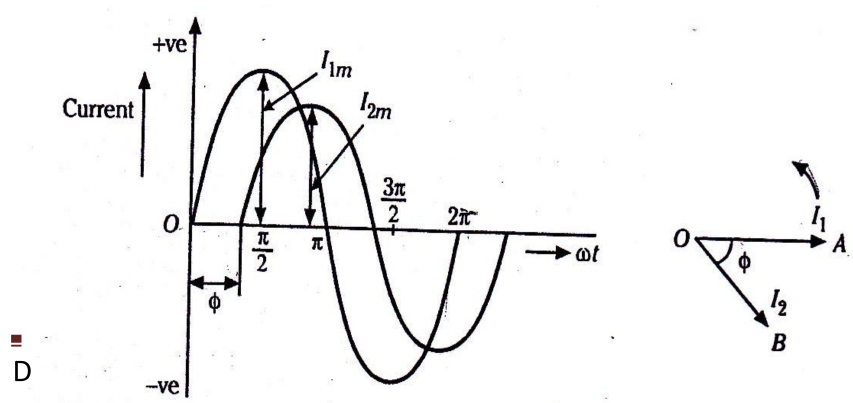


Fig. 3.8

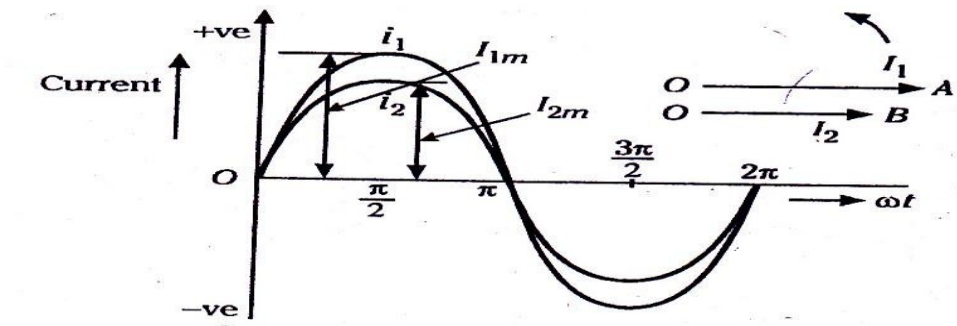


Fig. 3.9

The leading current I1 goes through its zero and maximum values first and the current I_2 goes through its zero and maximum values after time angle ϕ. The two waves representing these two currents are shown in Fig.3.8. if I_1 is taken as reference vector, two currents are expressed as

$$i_1 = I_{1m} \sin \omega t \qquad \text{and } i_2 = I_{2m} \sin (\omega t-\phi)$$

The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction, as shown in Fig.3.9. However, if the two quantities pass through zero values at the same instant but rise in opposite, as shown in Fig.3.10, they are said to be in phase opposition i.e., the phase difference is 180^0. When the two alternating quantities have a phase difference of 90^0 or $\pi/2$ radians they are said to be in quadrature.

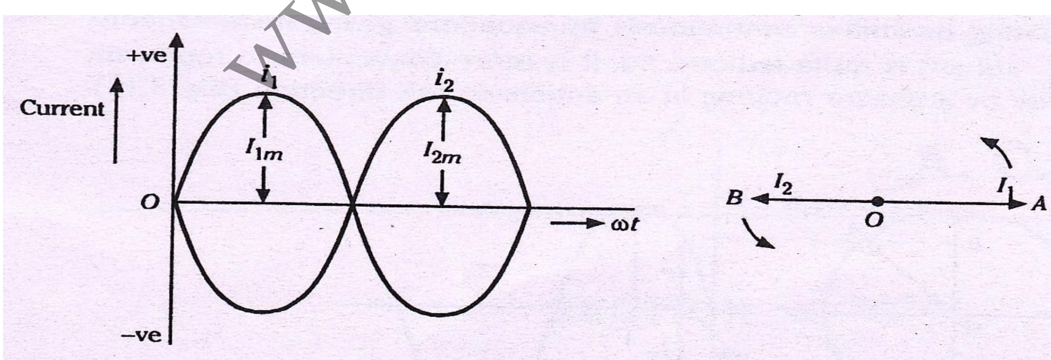


Fig. 3.10

Phasor Representation of Alternating Quantities

We know that an alternating voltage or current has sine waveform, and generators are designed to give e.m.f.s. with the sine waveforms. The method of representing alternating quantities continuously by equation giving instantaneous values (like e = Em sin ωt) is quite tedious. So, it is more convenient to represent a sinusoidal quantity by a phasor rotating in an anticlockwise direction (Fig.3.12).

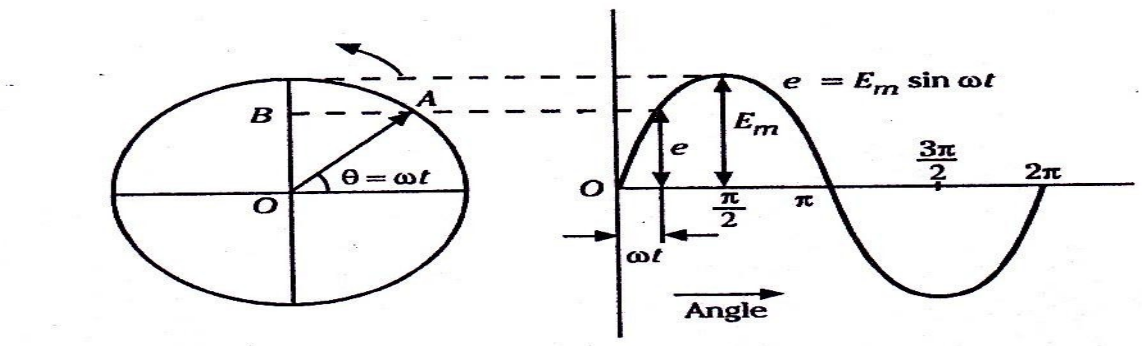


Fig. 3.12

While representing an alternating quantity by a phasor, the following points are to be kept in mind:

i) The length of the phasor should be equal to the maximum value of the alternating quantity.
ii) The phasor should be in the horizontal position at the alternating quantity is zero and is increasing in the positive direction.
iii) The inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.
iv) The angular velocity in an anti-clockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

Consider phasor 0A, which represents the maximum value of the alternating e.m.f. and its angle with the horizontal axis gives its phase (Fig.3.12). now, it will be seen that the projection of this phasor 0A on the vertical axis will give the instantaneous value of e.m.f.

∴ 0B = 0A sin ωt

Or e = 0A sin wt

= Em sin ωt

Note: The term „phasor" is also known as

„vector". a) 8+j6=$\sqrt{8^2+6^2}$ < tan 0.75=10<36.90

b) -10-j7.5=$\sqrt{(-10)^2+(-7.5)^2}$ < $\tan^{-1}$ 0.75

= 12.5 < $\tan^{-1}$ 0.75

This vector also falls in the third quadrant, so, following the same reasoning as mentioned in method 1, the angle when measured in CCW direction is

$= (180^0 + \tan^{-1} 0.75)$

$= 180^0 + 36.9^0 = 216.9^0$

Measured in CCW direct from + ve co-ordinate of x-axis, the angle is

- $(360^0 – 216.9^0) = - 143.1^0$

So this expression is written as

$12.5 < -143.1^0$ So, expression (ii) is

rewritten as

$10 < 36.9^0 \times 12.5 < -143.1^0$

$125 < -106.2^0$ which is the same as before