MODULE II

COPLANAR CONCURRENT AND NON CONCURRENT FORCE SYSTEM

1. **Four co-planar forces forces acting at a point are shown in fig Q3(a) . One of the forces is unknown and its magnitude is shown by 'P' . The resultant has a magnitude of 500N and is acting along the x-axis . Determine the unknown force 'P' and its inclination with x-axis.** **(Dec2014 /Jan 2015)**

$$\sum F_x = R = 500N$$

$$Pcos\theta = -791.59N$$

$$\sum F_y = 0$$

$$Psin\,\theta = 308.58\,N$$

$$\tan\theta = \frac{308.58}{791.59}$$

$$\theta = 21.29$$

$$\text{P} = \frac{308.58}{\sin 21.29}$$

P= 849.87KN

2. **State and prove Varignon's theorem of moments.** **(Dec2014 /Jan 2015)**

If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.

PROOF:

For example, consider only two forces F1 and F2 represented in magnitude and direction by AB and AC as shown in figure below.

Let O be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.

By the parallelogram law of forces, the diagonal AD represents, in magnitude and Direction, the resultant of two forces F1 and F2, let R be the resultant force. By geometrical representation of moments the moment of force about O=2 Area of triangle AOB the moment of force about O=2 Area of triangle AOC the moment of force about O=2 Area of triangle AOD

But,

Area of triangle AOD=Area of triangle AOC + Area of triangle ACD Also,

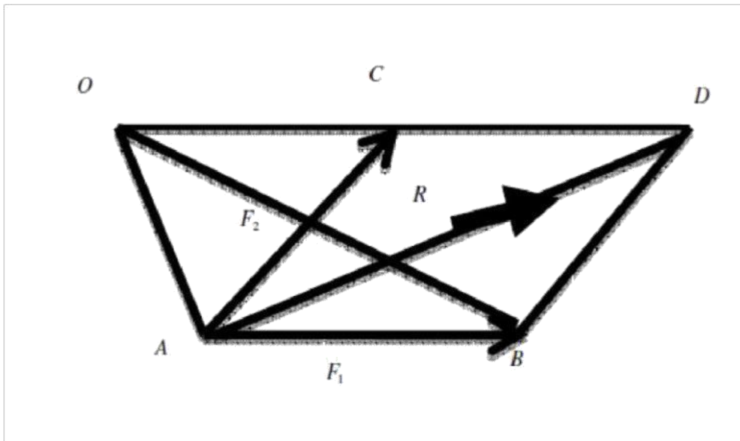
Area of triangle ACD=Area of triangle ADB=Area of triangle AOB Area of triangle AOD=Area of triangle AOC + Area of triangle AOB

Multiplying throughout by 2, we obtain

2 Area of triangle AOD =2 Area of triangle AOC+2 Area of triangle AOB

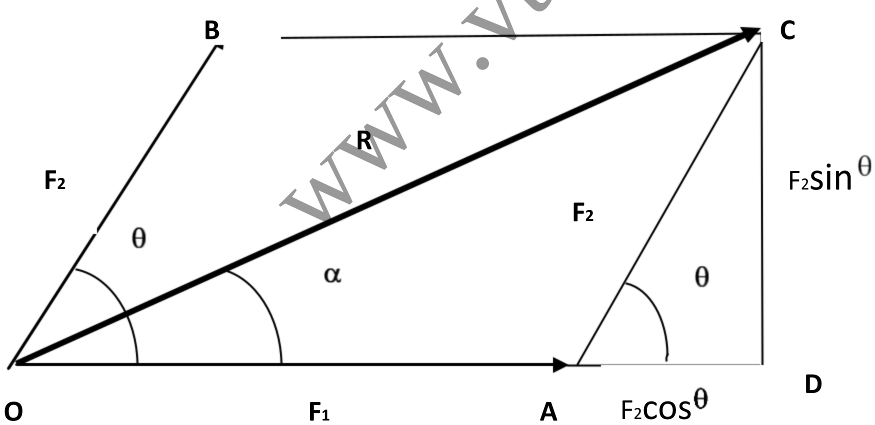
i.e., Moment of force R about O=Moment of force F1 about O + Moment of force F2 about O

Similarly, this principle can be extended for any number of forces.



3. State and prove parallelogram law of forces. **(Dec2014 /Jan 2015)**

<u>Parallelogram law of forces:</u> This law is applicable to determine the resultant of two coplanar concurrent forces only. This law states "***If two forces acting at a point are represented both in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant of the two forces is represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point.***"



Let F_1 and F_2 be two forces acting at a point O and θ be the angle between them. Let OA and OB represent forces F_1 and F_2 respectively both in magnitude and direction. The resultant R of F1 and F_2 can be obtained by completing a parallelogram with OA and OB as the adjacent sides of the parallelogram. The diagonal OC of the parallelogram represents the resultant R both magnitude and direction.

From the figure OC = $\sqrt{OD^2 + CD^2}$

$$= \sqrt{(OA + AD)^2 + CD^2}$$

$$= \sqrt{(F_1 + F_2 \cos\ )^2 + (F_2 \sin\theta\ )^2}$$

i.e $R = \sqrt{F_1^2 + F_2^2 + 2.\ F_1\ .\ F_2\ .\cos\theta}$ --------> 1

Let α be the inclination of the resultant with the direction of the F1, then

$$\alpha = \tan^{-1}\left[\frac{F_2 \sin\theta}{F_1 + F_2.\cos\theta}\right] \text{------- 2 } >$$

Equation 1 gives the magnitude of the resultant and Equation 2 gives the direction of the resultant.

4. Determine the magnitude, direction of the resultant force for the force system as shown in fig. Locate the resultant force with respect to point D. (Dec2014 /Jan 2015)

$$\sum F_x = 4 - 5\cos 26.56 = -0.472$$

$$\sum F_y = -6 - 5\sin 26.56 = -8.23$$

R = 8.24 KN

$\theta = 86.71°$

Take a moment about D

M_D= -8+4*1.2 -6*1 – 5 sin 26.56*2

M_D= -13.67 KN-m

D= 1.65m

5. **26KN force is the resultant of the forces, one of which is as shown in fig.Determine the other force. (Dec2014 /Jan 2015)**

R= 26KN

$\theta_1 = 67.38$

$\theta_2 = 36.87$

$R_x = \sum F_x$

26 cos 67.38 = 10 cos 36.86 +F cos θ

F cos θ = 2KN

$R_y = \sum F_y$

26 sin 67.38 =10 sin 36.86+ F sin θ

F sin θ = 18 KN

Dividing the equation

$$— = \frac{sin\theta}{cos\theta}$$

$$\theta = 83.65$$

$$F = \frac{18}{\sin 83.65} = 18.11\ KN$$

6. **Explain the principle of resolved parts. (Dec2014 /Jan 2015)**

$$\sum F_x = F_{x1} + F_{x2} + F_{x3} + F_{x4}$$

$\sum F_x = F_1 \, cos_{\theta 1} + \ F_2 cos_{\theta 2} + F_3 cos_{\theta 3} + F_4 cos_{\theta 4}$

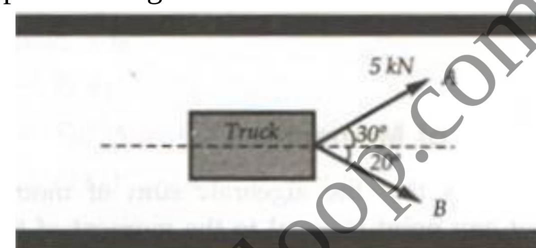
$\sum F_x = F_1 \, sin_{\theta 1} + \ F_2 sin_{\theta 2} + F_3 sin_{\theta 3} + F_4 sin_{\theta 4}$

$\theta = tan^{-1} \frac{\sum F_y}{\sum F_x}$

7. A truck is to be pulled along a straight road as shown in fig. (Jul/Aug 2013)

(i) If the force applied along rope A is 5kN inclined at 30^0, what should be the force in the rope B, which is inclined at 20^0, so that vehicle moves along the road?

(ii) If force of 4kN is applied in rope B at what angle rope B should be inclined so that the vehicle is pulled along the road?



If the vehicle is pulled along the road, the resultant force acting on it will be along the road. Taking x-axis along the road,

$\sum F_y = 0$

(i) $5\sin 30 - F_B \sin 20 =$

0 $F_B = 7.31$

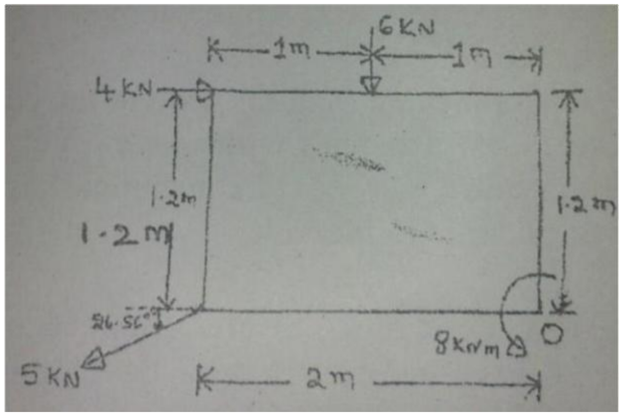
(ii) Let α be the angle made by rope B with X-axis

$\sum F_y = 0$

$5\sin 30 - 4 \sin \alpha = 0$ α

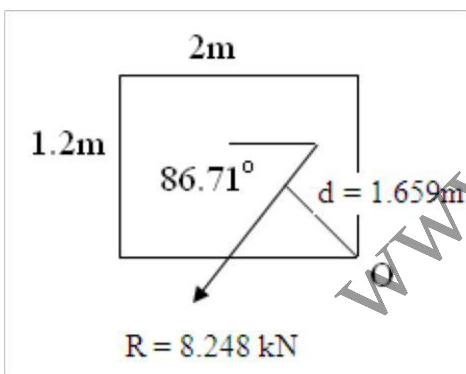
$= 36.68^o$

8. Determine the magnitude, direction of the resultant force for the force system shown in fig. Determine the X intercepts of the resultant force with respect to the point O. (Dec13)



$\sum F_X = 4 - 5\cos 26.56 = -0.472kN$

$\sum F_Y = -6 - 5\sin 26.56 = -8.236kN$
R = 8.248 kN α = tan-1
(8.236/0.472) = 86.71^o $\sum M_o = 4x1.2$
– 6x1 -8 - 5sin26.56 x 2
= -13.67kN m d
= 1.659m X intercept
= 1.659m
Y intercept = 28.96m



9. State and prove varignon's theorem **(Jan2013, June 2014)**

Varignon's principle of moments:

If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.

PROOF:

For example, consider only two forces F1 and F2 represented in magnitude and direction by AB and AC as shown in figure below.

Let O be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.

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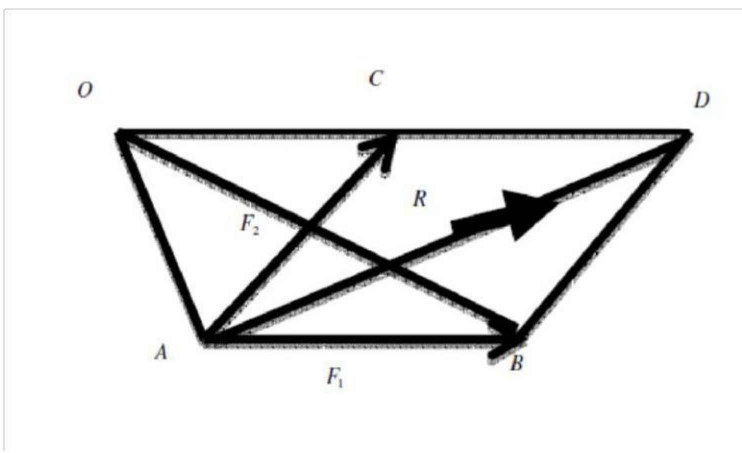
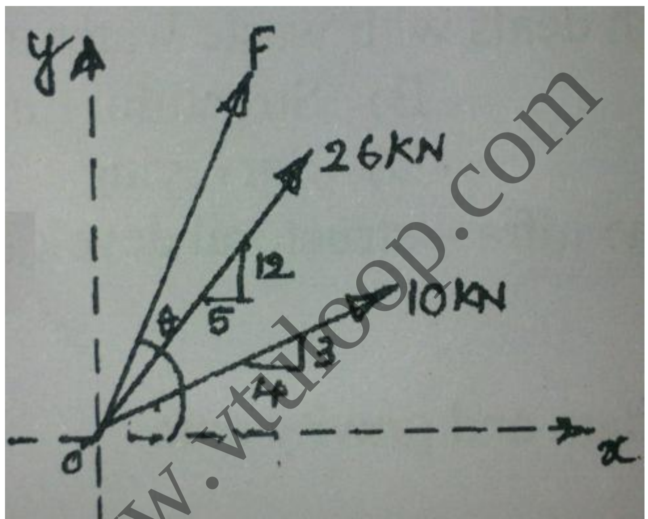


10. The 26kN force is the resultant of two forces, one of which is shown in fig. Determine the other force. (June 2012)



$\sum F_X = R \cos\alpha$

$F\cos\theta + 10\cos 36.86 = 26\cos 67.38$

$F\cos\theta = 2 \longrightarrow (1)$

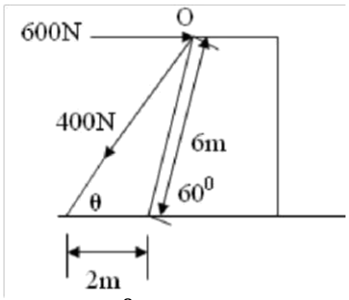
$\sum F_Y = R \sin\alpha$

$F\sin\theta + 10\sin 36.86 = 26\sin 67.38$

$F\sin\theta = 18 \longrightarrow (2)$

(1)/(2) tan θ
=9θ=
83.65 F =
18.11kN

11. Determine the resultant force acting on the structure at point O both in magnitude and direction for the system of forces shown in fig. (Jan 2013)



$\theta = 46.07^0$

$\sum F_X = 600 - 400\sin 46.07 = 322.49$

$\sum F_Y = -400\sin 46.07 = -288.07$

$R = 432.42N; \alpha = 41.77^0$

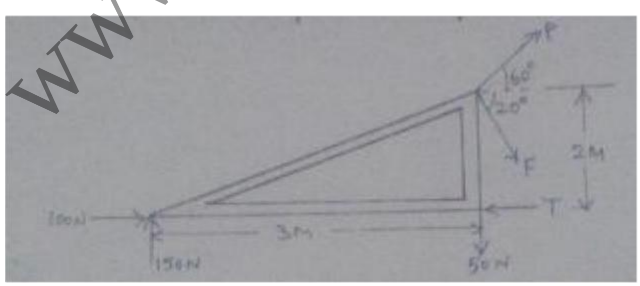
12. Two forces F_1 and F_2 act upon a body. If the magnitude of their resultant is equal to that of F1 and direction perpendicular to F_1, then find the magnitude and direction of force F_2. Take F_1 =20N **(June 2014)**

Sol: $20 - F2\cos\Theta = 0$

$\Theta = 45^o$

F2 = 28.28 N

13. Determine the forces P, F and T required to keep the frame in equilibrium **(June 2014)**



Sol: $\sum M_B = 0$

$-(100*2) + (150*3) + (T*2) = 0$

T=125N

$\sum Fx = 0$

$P\cos 60^o + F\cos 20^o - 125 + 100 = 0$

$\sum Fy = 0$

$P\sin 60^{o} - F\sin 20^{o} - 50 + 150 = 0$

$P = 104.162\ N$

$F = -28.81N$